

## Bilinear Model Approach for Cyclic and Dynamic Analysis of Semi-rigid Steel Frames

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### ABSTRACT

A comparison of the seismic behaviour of rigid and semi-rigid steel moment frames indicates that the seismic forces generated in a semi-rigid frames are sometimes less than the comparable rigid frames. The overall behaviour of such structures were mainly affected by the flexibility of the connections. To understand the real behaviour of steel semi-rigid frames, examination of the moment-rotation relationship of the beam-column connection must be undertaken.

The key parameters which influence the behaviour of the moment-rotation curve for this type of connection are identified. To investigate the influence of the flexibility of the connection on the overall behaviour of the frame, four semi-rigid steel frames with top and seat angle connection and double web angles were tested under cyclic and dynamic forces.

To investigate the key parameters of the connections and for verification of the proposed model, some of experimental monotonic moment-rotation curves are compared with this model. This is followed by a comparison of the experimental results obtained from cyclic and dynamic tests with analytical response. Analytical results, using the bilinear model approach in the non-linear finite element program ADAPTIC, are in good agreement with the results of monotonic, cyclic and dynamic tests.

### INTRDUCTION

Experimental results show that most moment-rotation curves for the top and seat angle connections with double web angles exhibit elastic-plastic behaviour with hardening characteristics. Therefore, based on this behaviour, for the design of semi-rigid steel frames and for simple implementation in the computer program a bilinear model for predicting the moment-rotation curve for the top and seat angle connection with double web angles is proposed in this study.

In the following sections, the method of evaluating the moment capacity, rotation and stiffness of top and seat angles with double web angles which is classified as a partial strength connection are described. Then, the experimental moment-rotation relationship are compared with analytical results.

### THEOREY AND ASSUMPTIONS

A typical top and seat angle connection with double web angles and its deformed pattern are shown in Figure 1. As indicated in this Figure, the vertical leg of the top angle attached to the column flange is pulled off the column surface by the beam and the outstanding leg of the seat angle is bent as a cantilever beam around the centre of rotation. Finally, a plastic failure mechanism is formed in the vertical leg of the angle attached to the flange of the column.

Figure 2 represents the collapse mechanism in the top angle. As shown in this Figure, two plastic hinges form. One of the hinges forms along the line of the nut edge of the fasteners and the second is located along the line of the toe of the fillet. To calculate the moment capacity of the connection, it is necessary to define first the location of the centre of rotation. An analysis of the available experimental research data indicated that most semi-rigid connections rotate about a point between the beam compression flange and the centre of gravity of the connection. This point then normally moves down to the beam compression flange as the connection undergoes inelastic behaviour. Therefore, the assumption that the centre of rotation is located in the compression flange of the beam can be used for calculating the moment capacity of the connection.

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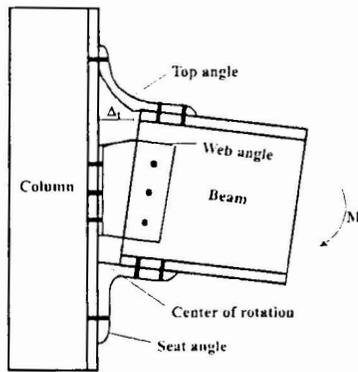


Figure 1

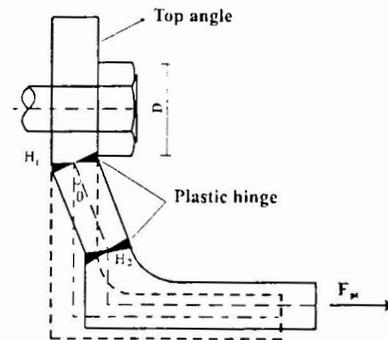


Figure 2

In order to consider yielding of the outstand leg of the angle for determining the moment capacity of the connection, it is important that the bolt slippage is avoided since this will cause a significant reduction in its ability to dissipate energy. This was observed from experimental results by Bernuzzi et al. (1992). For this purpose, the design philosophy should consist of sizing the bolt diameter and the pretensioning force such that yielding of the angle legs precedes bolt slip. In addition, it is assumed that the flexural stiffness of the web angle could be neglected in the evaluation of the moment capacity of the connection. The reason for this assumption is that the web angles are expected to carry a large proportion of the shear load. And since large moments are usually accompanied by large shear forces, shear connections are expected to yield in shear. Thus, the remaining area for carrying bending moments is very small and hence contribute little to the moment capacity of the connection. This behaviour was observed in the cyclic tests (Danesh, F., 1996).

By considering the theory and assumptions presented above,  $M_y$ ,  $\theta_y$ ,  $K_1$ ,  $K_2$ , and  $\theta_{max}$  should be determined in order to construct the bilinear moment-rotation model.

### Calculation of Yield Moment

As described in the previous section, when the connection responds in the inelastic range, two plastic hinges occur in the vertical leg of the top angle. The virtual work equation for the top angle at the collapse state is given by:

$$2 M_{pt} \theta = F_{pt} L \theta \quad (1)$$

where

- $M_{pt}$  = Plastic moment of the top angle
- $F_{pt}$  = Plastic shear force in the vertical leg
- $d_1$  = Distance from centre of the bolt to the heel of the angle
- $k$  = Distance from heel to toe of fillet of angle
- $D$  = Width of nuts across the flat sides for bolt fasteners
- $L$  = Distance between two plastic hinges (is given by  $d_1 - k - D/2$ )

To evaluate the shear force, the bending moment versus shear interaction should be considered in the analysis. This is because the distance between the two hinges is very short when compared with the thickness of the angle. For this purpose, the bending moment-shear interaction formula for the yield is used:

$$\frac{M_p}{M_o} + \left( \frac{F_{pt}}{F_o} \right)^2 = 1 \quad (2)$$

where  $M_o$  and  $F_o$  are the plastic bending moment and the plastic shear force without coupling and are determined from equations (3) and (4).

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$$M_o = \frac{\sigma_y l_t (t_t)^2}{4} \quad (3)$$

$$F_o = \frac{\sigma_y l_t t_t}{2} \quad (4)$$

in which  $\sigma_y$  is the yield stress.  $l_t$  is the length of the top angle and  $t_t$  is the thickness of the angle. By substituting  $M_p$  and  $M_o$  from the above equations into (2),  $F_{pt}$  can be determined, using an iterative procedure, from:

$$\left(\frac{F_{pt}}{F_o}\right)^4 + \frac{L}{t_t} \left(\frac{F_{pt}}{F_o}\right) = 1 \quad (5)$$

Having evaluated  $F_{pt}$ ,  $M_{pt}$  can then be calculated from Equation (1). The moment capacity contributed by the top angle  $M_{ut}$  is obtained by taking moments about the centre of rotation as follow:

$$M_{ut} = M_{pt} + F_{pt} d_2 \quad (6)$$

where  $d_2 = d_B + t_t$  and  $d_B$  is the depth of the beam.

To evaluate the moment capacity of the seat angle, it is assumed that the bearing pressure is uniformly distributed over the outstanding leg of the seat angle and thus the plastic moment capacity of the seat angle  $M_{us}$  is,

$$M_{us} = \frac{\sigma_y l_s (t_s)^2}{4} \quad (7)$$

$$M_{yc} = \frac{\sigma_y l_s (t_t)^2}{4} + \frac{F_{pt} L}{2} + F_{pt} (d_B + t_t) \quad (8)$$

where  $l_s$  and  $t_s$  are the length and the thickness of the seat angle respectively. By using Equations (6) and (7), the moment capacity of the top and seat angle connection with the double web angle can be obtained from Equation 8.

### Yield Rotation

The relationship between the deformation of the top and seat angles corresponding to the yielding of the connection and the thickness of the angles was developed by Harper et al. (1990). For a wide range of different angle thicknesses it was observed that the deformation of the angles was almost constant and approximately equal to 1.4 mm. This behaviour was noted in the experimental results of models CS1, CS2 and CS3 with different thickness of the top and seat angles ( Danesh, F., 1996).

Considering this value of deformation and making the assumption that the location of the centre of rotation is in the compression flange of the beam, the yield rotation can then be evaluated as shown in Equation (9):

$$\theta_y = \frac{1.4}{d_B} \quad (9)$$

where  $\theta_y$  is the yield rotation and  $d_B$  is the depth of the beam. In this Equation the unit of  $d_B$  is in mm.

### Evaluation of the Elastic and Post Elastic Stiffness

By determining the moment and rotation from equations (8) and (9), the elastic stiffness of the connection is obtained by dividing the yield moment by the yield rotation.

Experimental results for semi-rigid models subjected to cyclic tests indicate that the top and seat angles with double web angle connection exhibit significant post-elastic stiffness. As mentioned previously, in the inelastic

range larger forces of approximately twice the yield value were observed in the tests. Similar observation was also made in experimental studies carried out by Azizinamini et al. (1985). Such behaviour can be attributed to the material strain hardening.

From experimental moment-rotation curves, it was concluded that a value between 8% and 15% of the elastic stiffness reasonably represents the post-elastic stiffness. According to the results and observations from the moment-rotation curves (Danesh, F., 1996) a value of 10% of the elastic stiffness is recommended for a bilinear representation of the connection response.

### Maximum rotation

According to studies carried out by researchers (Danesh, F., 1996), the maximum rotation of the top and seat angle connection with double web angle should not exceed 0.05 radians. Therefore, the value of 0.04 radians observed from the tests is selected as a maximum rotation of the connection in analysis of semi-rigid steel frames.

## COMPARISON WITH EXPERIMENTAL RESULTS

In this section, some of the experimental monotonic moment-rotation curves are compared with the proposed bilinear model. This is followed by a comparison of the experimental results (from cyclic and dynamic tests) with the analytical response using the proposed model.

### Monotonic Tests

In order to compare the results of the proposed bilinear model with the tests results, it is instructive to first examine the monotonic moment-rotation curves obtained from cyclic tests. Furthermore, the accuracy of the model is checked (for changes in the key parameters of the connection) via comparisons with the results of experimental tests carried out by Azizinamini et al. (1985) and Danesh (1996). Details of these connections are given in references.

The experimental moment-rotation relationships of the two models (8S5 and CS3) are shown in Figures 3 and 4. These Figures confirm that there is good agreement between the model prediction of the moment-rotation relationship and the experimental results. This agreement is even more significant in the post-elastic region of the curves. It is therefore concluded that the proposed bilinear moment-rotation curve represents with reasonable accuracy the yield and ultimate values and hence it may be used with confidence.

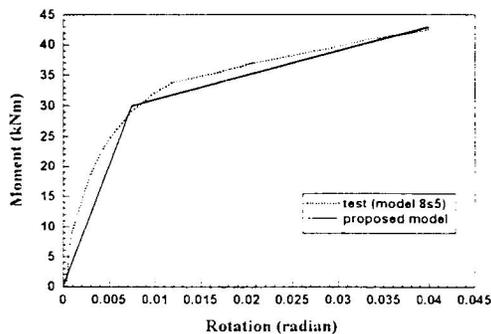


Figure 3

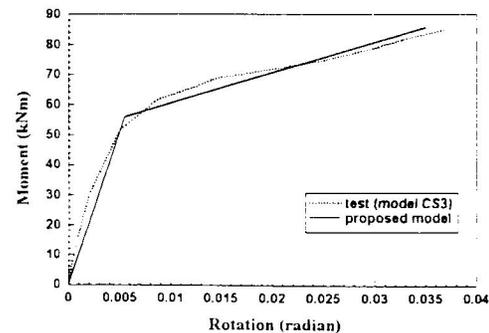


Figure 4

### Cyclic Tests

Figures 5 and 6 show the comparison between the analytical and experimental data for two specimens (CS2 and CS3) relating to the second storey restoring forces and the base shears versus the top storey displacements. As

indicated in these Figures, the hysteretic behaviour of the curve for all the models is in good agreement with the tests in terms of force and displacement.

Table 1 demonstrates the differences between the analytical and experimental yield and the ultimate parameters. The maximum 3% interstorey drift has been identified as an ultimate limit state for each model. As Table 1 indicates, the maximum discrepancy between the values obtained from the analyses to the values observed in the tests is less than 6%.

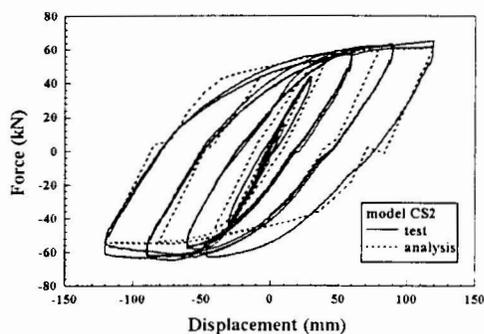


Figure 5

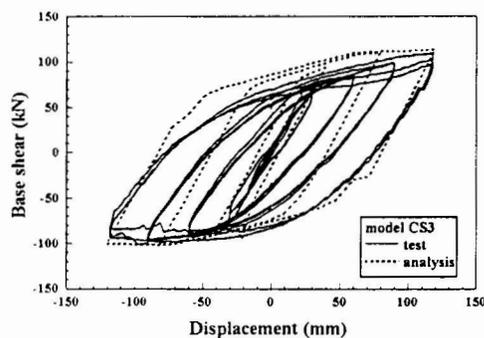


Figure 6

**Table 1**  
Comparison of analytical yield and ultimate values with experimental results

Specimens	Method	Yield displacement (mm)	Yield base shear (kN)	Ultimate base shear (kN)	Ultimate Moment of col. (kNm)
CS1	Test	30	45	70	145.94
	Analysis	30.59	47.7	70.47	149.32
	Differences (%)	+ 2	+ 6	+ 0. 67	+ 2.3
CS2	Test	26	60	94	161.83
	Analysis	27.5	61.2	95.5	158.20
	Differences (%)	+ 5.8	+ 2	+ 1.5	- 2.24
CS3	Test	24	65	104.5	162
	Analysis	23.70	68.39	110.82	160.13
	Differences (%)	- 1.25	+ 5.20	+ 6	- 1.15

It is also observed that the analytical yield and the ultimate parameters in the three models are in good agreement with the test results. This therefore engenders further confidence in the use of bilinear model approach for the analysis of semi-rigid steel frames.

### Dynamic Tests

The experimental results and analytical simulations for model DS1 which was tested using the El-Centro earthquake record are given in Figures 7 and 8. Figure 7 shows the displacement response history obtained from experimental and analytical investigations whilst the comparison for the experimental and analytical base shear response is presented in Figure 8. Good agreement is observed between the experimental and analytical results. The analytical displacement response shows good agreement with the results of the test especially during the first 2.5 seconds of the record. The discrepancies observed when the model has achieved its maximum displacement may be attributed to the cumulative errors of the pseudo-dynamic test.

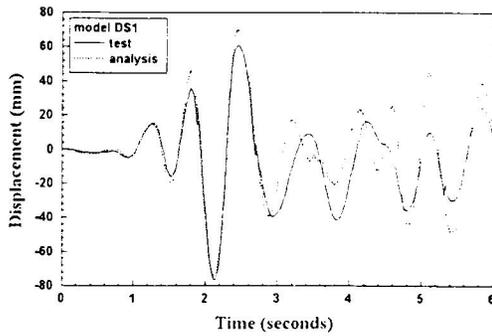


Figure 7

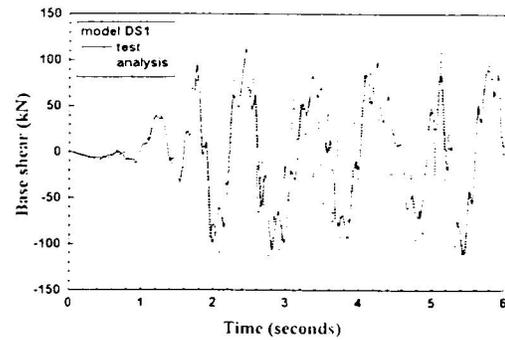


Figure 8

The yield and maximum time history response of this model are compared with the values obtained from the test in Table 2. As indicated in the Table, the predicted yield and maximum response are in good agreement with the test values, with differences less than 5 %.

Table 2  
Dynamic response of model DS1

Method	Record	Acceleration P.G.A.	Yield displacement	Maximum displacement	Maximum base shear
Test	El Centro	0.4 g	23 mm	76 mm	110
Analysis	El Centro	0.4 g	24 mm	73 mm	115
Differences	-	-	+ 4 %	- 4 %	+ 4.5 %

In general, both the displacement response and the shear forces show very good correlation between the pseudo-dynamic test results and the non-linear dynamic analysis simulations.

## CONCLUSIONS

In this research, an analytical bilinear model for predicting the moment-rotation curve of the top and seat angles with double web angles connection is presented. The key parameters which influence the behaviour of the moment-rotation curve for this type of connection are identified. Analytical results, using the proposed bilinear model in the non-linear finite element program are in good agreement with the results of monotonic, cyclic and dynamic tests. It is therefore concluded that by incorporating the proposed bilinear model into the non-linear finite element program it can be used as an effective tool for the inelastic analysis of semi-rigid steel frames with top and seat angles with double web angles connections. Consequently, this program is utilised for carrying out the parametric study to examine the real behaviour of semi-rigid steel frames and the influence of the flexibility of this type of connection on the overall behaviour.

## REFERENCES

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